Structural Mechanics

12.6 Forced vibration of system with two DOF under harmonic loading (两个自由度体系在简谐荷载下的强迫振动)

内容(Contents):

1. **概念(Concept)**：动力放大系数、振幅参数，荷载频率参数。
2. **理论(Theory)**：牛顿第二定律，和线性方程组的求解
3. **应用(Application)**：减震或者吸振器原理（**TMD**）

要求(Requirements):

运动方程的建立（刚度法和柔度法）；熟练动力放大系数的求法；理解减震或者吸振器的原理。

作业(Homework): 12-22, 23,24 。

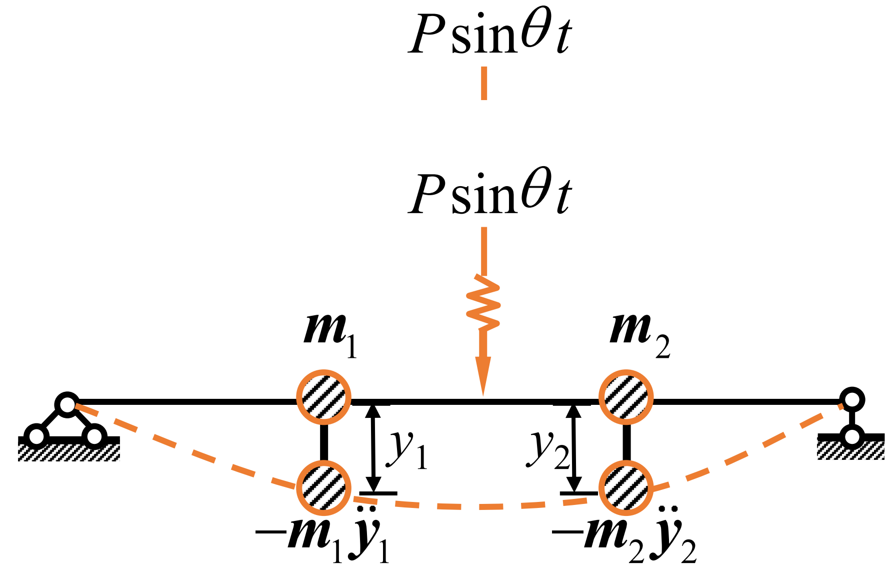
12.6 The forced vibration of system with two DOF under harmonic loading (两个自由度体系在简谐荷载下的强迫振动)

1. Flexibility method (柔度法)

2. Stiffness method (刚度法)

1. Flexibility method

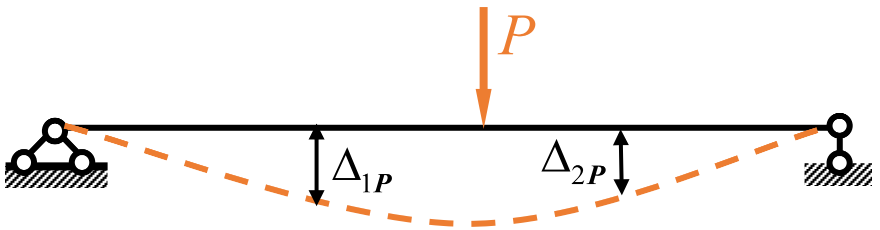
No damping



(1) To establish dynamic equation

Displacement equation

$$\begin{cases} y_1 = (-m_1\ddot{y}_1)\delta_{11} + (-m_2\ddot{y}_2)\delta_{12} + \Delta_{1P}\sin\theta t \\ y_2 = (-m_1\ddot{y}_1)\delta_{21} + (-m_2\ddot{y}_2)\delta_{22} + \Delta_{2P}\sin\theta t \end{cases}$$



$$\begin{cases} m_1\ddot{y}_1\delta_{11} + m_2\ddot{y}_2\delta_{12} + y_1 = \Delta_{1P}\sin\theta t \\ m_1\ddot{y}_1\delta_{21} + m_2\ddot{y}_2\delta_{22} + y_2 = \Delta_{2P}\sin\theta t \end{cases}$$

Particular solution：

$$\left.\begin{aligned} y_1(t) &= Y_1 \sin\theta t \\ y_2(t) &= Y_2 \sin\theta t \end{aligned}\right\}$$

(2) The solution of the dynamic equation

Homogeneous solution (齐次解) (ω_r)

\+ Particular solution (特解) (θ)

$$\left.\begin{aligned} y_1(t) &= Y_1 \sin\theta t \\ y_2(t) &= Y_2 \sin\theta t \end{aligned}\right\} \Rightarrow \begin{aligned} m_1\ddot{y}_1\delta_{11} + m_2\ddot{y}_2\delta_{12} + y_1 &= \Delta_{1P}\sin\theta t \\ m_1\ddot{y}_1\delta_{21} + m_2\ddot{y}_2\delta_{22} + y_2 &= \Delta_{2P}\sin\theta t \end{aligned}$$

Solution:

$$(\boldsymbol{m}_1\theta^2\delta_{11} - 1)\boldsymbol{Y}_1 + \boldsymbol{m}_2\theta^2\delta_{12}\boldsymbol{Y}_2 + \Delta_{1\boldsymbol{P}} = 0$$

$$\boldsymbol{m}_1\theta^2\delta_{21}\boldsymbol{Y}_1 + (\boldsymbol{m}_2\theta^2\delta_{22} - 1)\boldsymbol{Y}_2 + \Delta_{2\boldsymbol{P}} = 0$$

Amplitude: $\boldsymbol{Y}_1 = \dfrac{\boldsymbol{D}_1}{\boldsymbol{D}_0} \quad \boldsymbol{Y}_2 = \dfrac{\boldsymbol{D}_2}{\boldsymbol{D}_0}$

$$\boldsymbol{D}_0 = \begin{vmatrix} (\boldsymbol{m}_1\theta^2\delta_{11} - 1) & \boldsymbol{m}_2\theta^2\delta_{12} \\ \boldsymbol{m}_1\theta^2\delta_{21} & (\boldsymbol{m}_2\theta^2\delta_{22} - 1) \end{vmatrix}$$

$$\boldsymbol{D}_1 = \begin{vmatrix} -\Delta_{1\boldsymbol{P}} & \boldsymbol{m}_2\theta^2\delta_{12} \\ -\Delta_{2\boldsymbol{P}} & (\boldsymbol{m}_2\theta^2\delta_{22} - 1) \end{vmatrix} \qquad \boldsymbol{D}_2 = \begin{vmatrix} (\boldsymbol{m}_1\theta^2\delta_{11} - 1) & -\Delta_{1\boldsymbol{P}} \\ \boldsymbol{m}_1\theta^2\delta_{21} & -\Delta_{2\boldsymbol{P}} \end{vmatrix}$$

Discussion

$$D_0 = \begin{vmatrix} (m_1\theta^2\delta_{11}-1) & m_2\theta^2\delta_{12} \\ m_1\theta^2\delta_{21} & (m_2\theta^2\delta_{22}-1) \end{vmatrix} \quad D_1 = \begin{vmatrix} -\Delta_{1P} & m_2\theta^2\delta_{12} \\ -\Delta_{2P} & (m_2\theta^2\delta_{22}-1) \end{vmatrix} \quad D_2 = \begin{vmatrix} (m_1\theta^2\delta_{11}-1) & -\Delta_{1P} \\ m_1\theta^2\delta_{21} & -\Delta_{2P} \end{vmatrix}$$

1)when $\theta \to 0$ Static loading

Amplitude: $Y_1 = \dfrac{D_1}{D_0}$ $Y_2 = \dfrac{D_2}{D_0}$

$$\mathrm{D}_0 \to 1,\ \mathrm{D}_1 \to \Delta_{1P},\ \mathrm{D}_2 \to \Delta_{2P} \quad \Rightarrow Y_1 \to \Delta_{1P},\ Y_2 \to \Delta_{1P}$$

2)when $\theta \to \infty$ No time to react

$$\mathrm{D}_0 \propto \theta^4, \mathrm{D}_1 \propto \theta^2, \mathrm{D}_2 \propto \theta^2 \quad \Rightarrow Y_1 \to 0,\ Y_2 \to 0$$

Discussion

$$D_0 = \begin{vmatrix} (m_1\theta^2\delta_{11}-1) & m_2\theta^2\delta_{12} \\ m_1\theta^2\delta_{21} & (m_2\theta^2\delta_{22}-1) \end{vmatrix} \quad D_1 = \begin{vmatrix} -\Delta_{1P} & m_2\theta^2\delta_{12} \\ -\Delta_{2P} & (m_2\theta^2\delta_{22}-1) \end{vmatrix} \quad D_2 = \begin{vmatrix} (m_1\theta^2\delta_{11}-1) & -\Delta_{1P} \\ m_1\theta^2\delta_{21} & -\Delta_{2P} \end{vmatrix}$$

Amplitude: $Y_1 = \frac{D_1}{D_0} \quad Y_2 = \frac{D_2}{D_0}$

3)when $\theta = \omega_1$ or $\theta = \omega_2$

$$D_0 = \begin{vmatrix} (m_1\omega^2\delta_{11}-1) & m_2\omega^2\delta_{12} \\ m_1\omega^2\delta_{21} & (m_2\omega^2\delta_{22}-1) \end{vmatrix}$$

$D_0=0$ and not both D_1, D_2 are zero (不全为零时)

$\Rightarrow Y_1 \to \infty,\ Y_2 \to \infty$

Resonance
(共振)