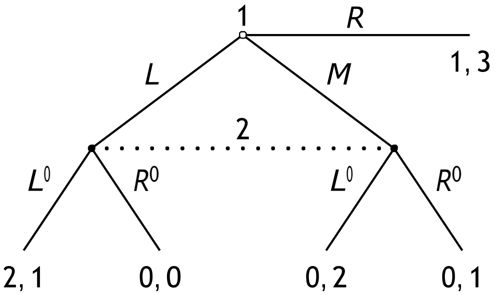
Dynamic Games of Incomplete Information

Bo Shen

Economics and Management School, Wuhan University

June, 2016

- Example 1:



- What are the pure-strategy Nash equilibria and subgame-perfect Nash equilibria in this game?

- The normal-form representation of the game:

		Player 2 L^0	Player 2 R^0
Player 1	L	2, 1	0, 0
Player 1	M	0, 2	0, 1
Player 1	R	1, 3	1, 3

- Two pure-strategy Nash equilibria:

$$(L, L^0), \text{ and } (R, R^0)$$

- Since the above game has no subgames, both (L, L^0) and (R, R^0) are subgame-perfect Nash equilibria

Introduction

- However, (R, R^0) is based on a non-credible threat:
  - On one hand, if player 1 believes player 2's threat of playing R^0, then player 1 indeed should choose R to end the game with payoff 1, which is larger than 0 by choosing L or M
  - On the other hand, if player 1 doesn't believe the threat and plays L or M, then when player 2 gets the move, he will choose L^0, since L^0 strictly dominates R^0 for player 2
- The threat of playing R^0 from player 2 is not credible
- We need to strengthen the equilibrium concept to rule out some subgame-perfect Nash equilibria like (R, R^0)
- A stronger equilibrium concept $\Rightarrow$ perfect Bayesian equilibrium

Requirement 1

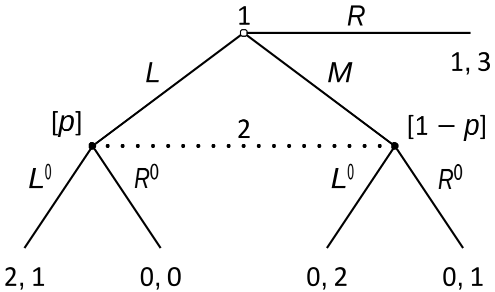
At each information set, the player with the move must have a belief about which node in the information set has been reached by the play of the game. For a non-singleton information set, a belief is a probability distribution over the nodes in the information set; for a singleton information set, a belief puts probability one on the single decision node.

Perfect Bayesian Equilibrium

Requirement 2

Given their beliefs, the players' strategies must be sequentially rational. That is, at each information set, the action taken by the player with the move (and the player's subsequent strategy) must be optimal, given the player's belief at that information set and the other players' subsequent strategies (where a subsequent strategy is a complete plan of action covering every contingency that might arise after the given information set has been reached).

- In Example 1, by Requirement 1, if player 2's nonsingleton information set is reached, player 2 must form a belief on which of the decision node has been reached, i.e., player 2 believes that player 1 has chosen L with probability p, and M with probability $1 - p$



- Given this belief, player 2's expected payoffs are
  - playing L^0: $p \bullet 0 + (1-p) \bullet 2 = 2-p$
  - playing R^0: $p \bullet 0 + (1-p) \bullet 1 = 1-p$
- Since R^0 is never optimal for any belief, (R, R^0) cannot satisfy Requirement 2
- Requirements 1 and 2 together can already eliminate the equilibrium (R, R^0) which relies on a non-credible threat
- Requirements 1 and 2 allow for arbitrary beliefs, including unreasonable ones. Further requirements on players' beliefs need to be introduced

Definition

For a given equilibrium in a given extensive-form game, an information set is **on the equilibrium path** if it will be reached with positive probability if the game is played according to the equilibrium strategies, and is **off the equilibrium path** if it is definitely not to be reached if the game is played according to the equilibrium strategies.

- Here the "equilibrium" can mean Nash equilibrium, subgame perfect Nash equilibrium, Bayesian Nash equilibrium or perfect Bayesian Nash equilibrium

- In Example 1, for the equilibrium (L, L^0), the player 2's nonsingleton information set is on the equilibrium path, while there is no information set off the equilibrium path
- For the equilibrium (R, R^0), the player 2's nonsingleton information set is off the equilibrium path, and there is no information set on the equilibrium path

Requirement 3

At information sets on the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies.

- In Example 1, suppose the equilibrium (L, L^0) is played, then Requirement 3 implies player 2's belief must be $p = 1$
- Consider a hypothetical situation: the game has a mixed-strategy equilibrium in which player 1 plays L with probability q_1, M with probability q_2, and R with probability $1 - q_1 - q_2$. Requirement 3 would force player 2's belief to be

$$p = \text{Prob}(L \text{ is played } | L \text{ or } M \text{ is played}) = \frac{q_1}{q_1 + q_2}$$

Requirement 4

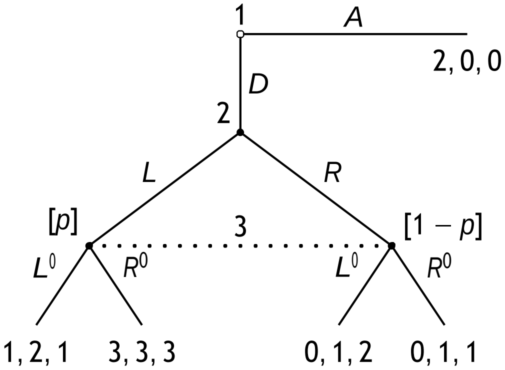
At information sets off the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies where possible.

- In Example 1, suppose the equilibrium $(\boldsymbol{R}, \boldsymbol{R}')$ is played, for player 2's nonsingleton information set that is off the equilibrium path, player 2's belief $\boldsymbol{p}$ is not restricted by Requirement 4

Definition

A perfect Bayesian equilibrium consists of strategies and beliefs satisfying Requirements 1 through 4.

- Example 2:



- What are the (pure-strategy) Nash equilibria and subgame-perfect Nash equilibria of this game? Are they also perfect Bayesian equilibria?