Structural Mechanics

15.1 Free vibration of multi-DOF system(多自由度体系的自由振动)

内容(Contents):

1. **概念(Concept)：** 振型正交性、振型归一化。
2. **理论(Theory)：** 牛顿第二定律，和线性方程组的求解
3. **应用(Application)：** 高层建筑或者工程结构的频率与振型的计算

要求(Requirements):

运动方程的建立（刚度法和柔度法）； 熟练刚度系数和柔度系数的计算；多自由度体系的固有频率和振型的计算；理解正交性的物理意义。

作业(Homework): 15-1, 15-2 。

15.1 Free vibration of multi-DOF system

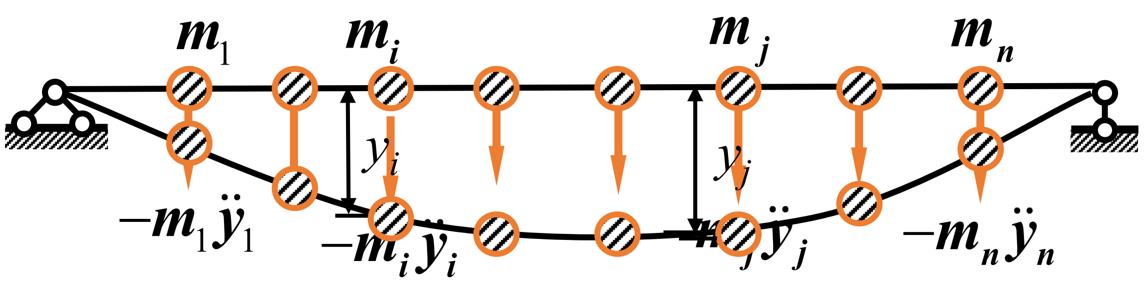
15.1.2 Flexibility method

15.1.1 Stiffness method

15.2 The orthogonality of the principle modes of vibration (主振型的正交性)

15.1.2 Flexibility method

The dynamic displacements of the mass points are induced by inertia force



Displacement equation：

$$\left.\begin{aligned}
y_1 &= \delta_{11}(-m\ddot{y}_1)+\delta_{12}(-m\ddot{y}_2)+\cdots\delta_{1n}(-m\ddot{y}_n)\\
y_2 &= \delta_{21}(-m\ddot{y}_1)+\delta_{22}(-m\ddot{y}_2)+\cdots\delta_{2n}(-m\ddot{y}_n)\\
&\cdots\cdots\\
y_n &= \delta_{n1}(-m\ddot{y}_1)+\delta_{n2}(-m\ddot{y}_2)+\cdots\delta_{nn}(-m\ddot{y}_n)
\end{aligned}\right\}$$

$$y_i = \sum_{j=1}^{n}\delta_{ij}(-m\ddot{y}_j) \quad (i=1,2,\cdots,n)$$

$$\left.\begin{aligned} \boldsymbol{y}_1 &= \delta_{11}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_1) + \delta_{12}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_2) + \cdots \delta_{1\boldsymbol{n}}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_{\boldsymbol{n}}) \\ \boldsymbol{y}_2 &= \delta_{21}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_1) + \delta_{22}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_2) + \cdots \delta_{2\boldsymbol{n}}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_{\boldsymbol{n}}) \\ &\cdots\cdots \\ \boldsymbol{y}_{\boldsymbol{n}} &= \delta_{\boldsymbol{n}1}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_1) + \delta_{\boldsymbol{n}2}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_2) + \cdots \delta_{\boldsymbol{nn}}(-\boldsymbol{m}\ddot{\boldsymbol{y}}_{\boldsymbol{n}}) \end{aligned}\right\}$$

⇓

$$\begin{Bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \vdots \\ \boldsymbol{y}_{\boldsymbol{n}} \end{Bmatrix} + \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1\boldsymbol{n}} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2\boldsymbol{n}} \\ \vdots & \vdots & \vdots & \vdots \\ \delta_{\boldsymbol{n}1} & \delta_{\boldsymbol{n}2} & \cdots & \delta_{\boldsymbol{nn}} \end{bmatrix} \begin{bmatrix} \boldsymbol{m}_1 & & & \\ & \boldsymbol{m}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{m}_{\boldsymbol{n}} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{y}}_1 \\ \ddot{\boldsymbol{y}}_2 \\ \vdots \\ \ddot{\boldsymbol{y}}_{\boldsymbol{n}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

or： $\{\boldsymbol{y}\} + [\delta][\boldsymbol{M}]\{\ddot{\boldsymbol{y}}\} = \{0\}$

Dynamic equation: $\{y\} + [\delta][M]\{\ddot{y}\} = \{0\}$

Solution: $\{y\} = \{Y\}\sin(\omega t + \alpha)$

$$\{Y\}\,Sin(\omega t + \alpha) - \omega^2[\delta][M]\{Y\}\,Sin(\omega t + \alpha) = \{0\}$$

$$\left([\delta][M] - \lambda\ [I]\right)\{Y\} = \{0\}$$

$$\left([\delta][M]-\lambda\,[I]\right)\{Y\}=\{0\}$$

Non-zero solution

$$\left|[\delta][M]-\lambda[I]\right|=\{0\}$$

Frequency equation

Equation of λ with n degrees

(关于 λ 的n 次方程)

$$\begin{vmatrix} (\delta_{11}m_1-\lambda) & \delta_{12}m_2 & \cdots & \delta_{1n}m_n \\ \delta_{21}m_1 & (\delta_{22}m_2-\lambda) & \cdots & \delta_{2n}m_n \\ \cdots & \cdots & \cdots & \cdots \\ \delta_{n1}m_1 & \delta_{n2}m_2 & \cdots & (\delta_{nn}m_n-\lambda) \end{vmatrix}=0$$

$\Rightarrow \lambda_1,\lambda_2,\cdots\lambda_n$

$\Rightarrow \omega_1,\omega_2,\cdots\omega_n$

principle modes of vibration:

$$\left([\delta][M]-\lambda_i[I]\right)\{Y\}^{(i)}=\{0\} \Rightarrow \{Y\}^{(1)},\{Y\}^{(2)},\cdots,\{Y\}^{(n)}$$