Instrumentation and Control system

Lecture 10

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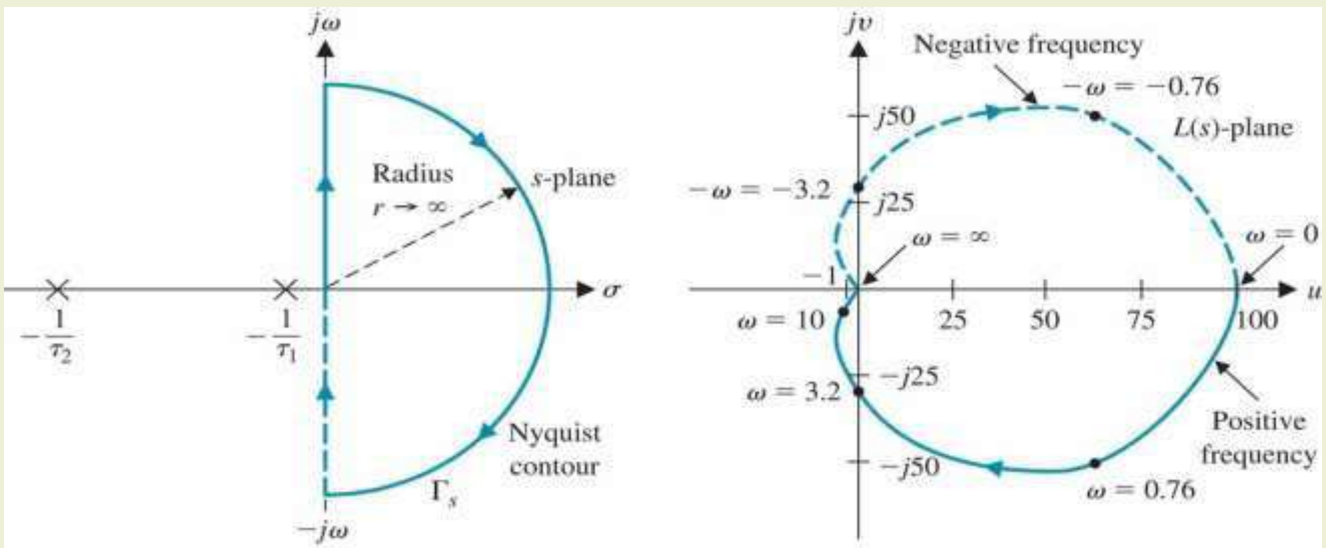
Contents

- *Experimental and theoretical determination of frequency response with phase margin and gain margin.*
- *Stability analysis using Bode plot, and Nyquist stability criterion.*
- *Controller design using frequency response method, Phase-lead and Phase-lag controls.*
- *Full state feedback control design.*

Example 1

$$L(s) = \frac{100}{(s+1)(0.1s+1)}$$

Number of poles of $L(s)$ in the right hand s-plane is zero, thus $P = 0$, therefore system is stable. We require $N = Z = 0$, and contour must not encircle (-1,0) point in $L(s)$-plane.



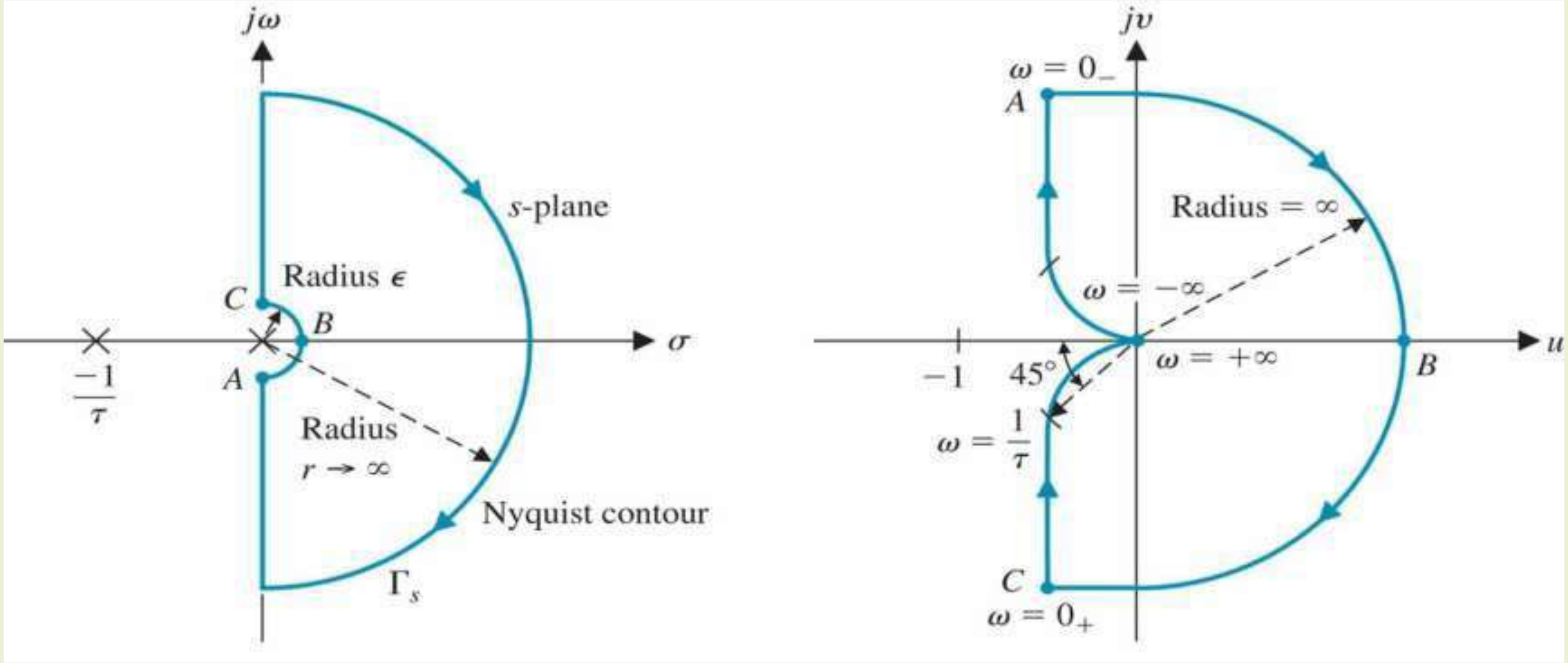
ω	0	0.1	0.76	1	2	10	20	100	∞
$\lvert L(j\omega)\rvert$	100	96	79.6	70.7	50.2	6.8	2.24	0.10	0
$\angle L(j\omega)$ (degrees)	0	−5.7	−41.5	−50.7	−74.7	−129.3	−150.5	−173.7	−180

Example 2

$$L(s) = \frac{K}{s(\tau s+1)},$$ Poles of $L(s)$ at origin and $-1/\tau$.

1. Origin of s-plane, $\omega = 0_-$ to $\omega = 0_+$
2. Portion from $\omega = 0_+$ to $\omega = +\infty$
3. Portion from $\omega = +\infty$ to $\omega = -\infty$
4. Portion from$\omega = -\infty$ to $\omega = 0_-$

$P = 0$ within RHP ➔ stable ➔ $N = Z = 0$ ➔ Γ_L must not encircle the (-1,0)

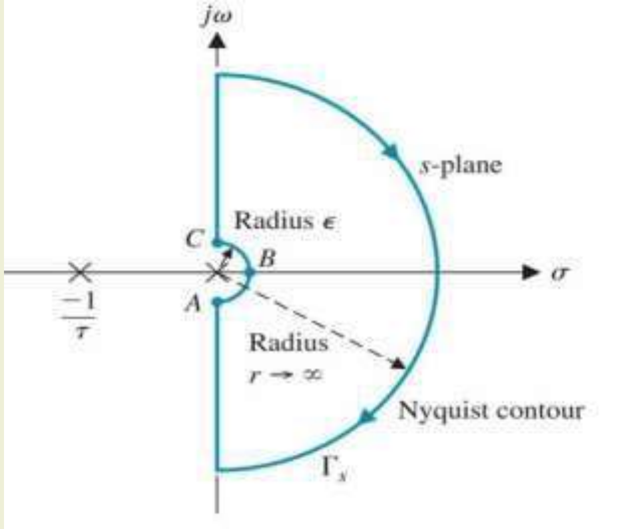


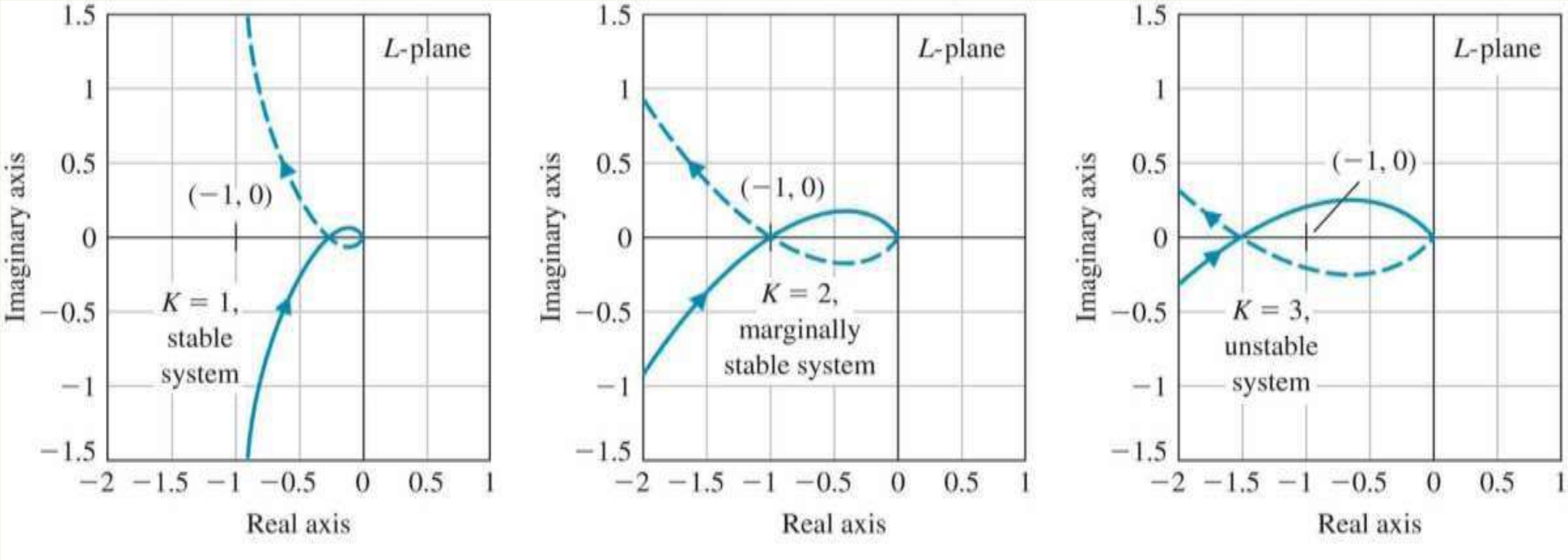
Example 3

$L(s) = \frac{K}{s(\tau_1 s+1)(\tau_2 s+1)}$, Poles of $L(s)$ at origin, $-1/\tau_1$ and $-1/\tau_2$

v in (9.26) and *u* in (9.27)

System is stable when $\frac{-K\tau_1\tau_2}{\tau_1+\tau_2} \geq -1$





Example 4

$$L(s) = \frac{K}{s^2(\tau s+1)}$$

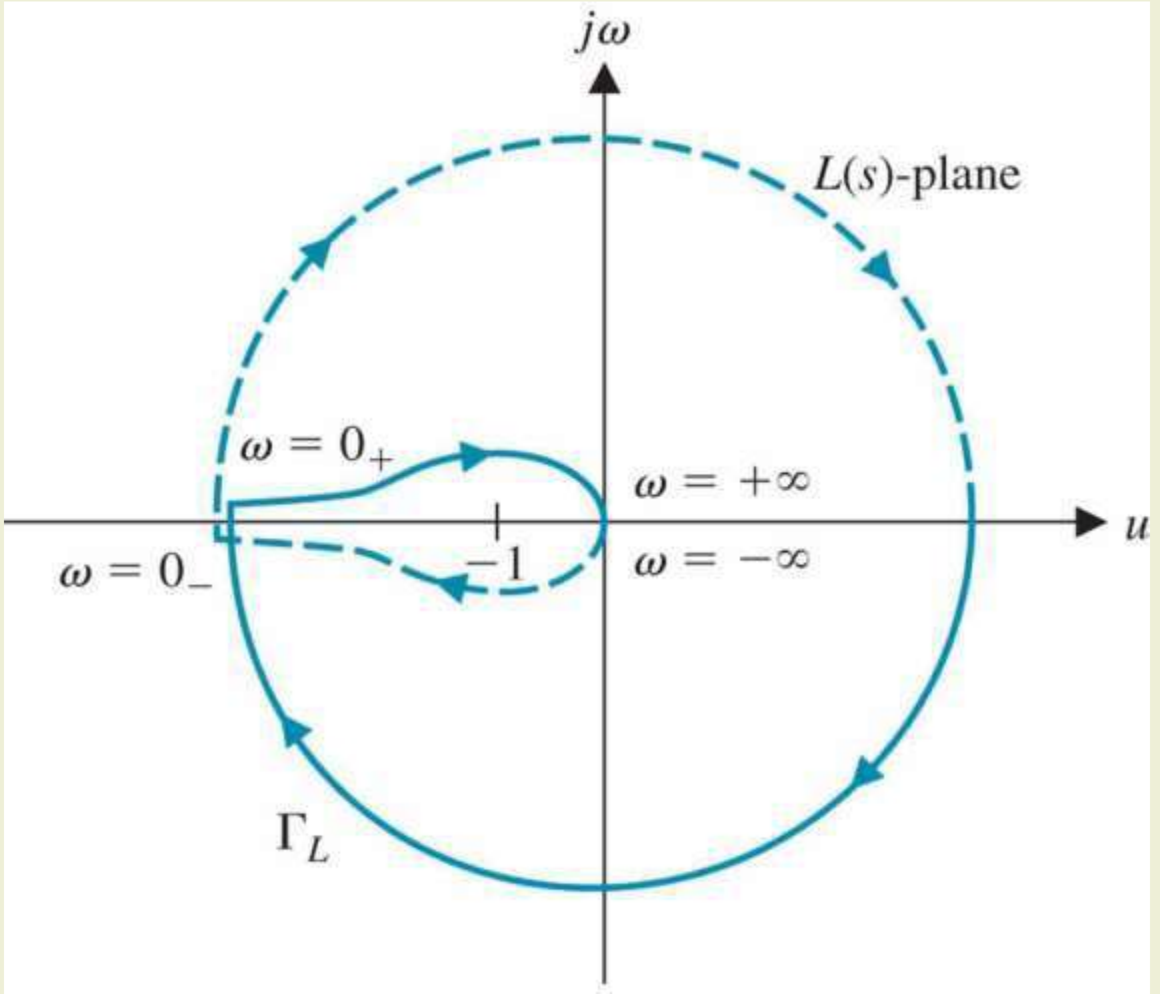
$$L(j\omega) = \frac{K}{-\omega^2(j\omega\tau+1)} = \frac{K}{[\omega^4+\tau^2\omega^6]^{1/2}} \angle - \pi - tan^{-1}(\omega\tau).$$

Angle of $L(j\omega)$ is always -180^o or less, and the locus of $L(j\omega)$ is above

1. As ω approaches to 0_+
   $$\lim_{\omega\to 0_+} L(j\omega) = \lim_{\omega\to 0_+} \left|\frac{K}{\omega^2}\right| \angle - \pi$$
2. As ω approaches to $+\infty$
   $$\lim_{\omega\to +\infty} L(j\omega) = \lim_{\omega\to +\infty} \left|\frac{K}{\omega^3}\right| \angle - 3\pi/2$$
3. At small semicircular detour at the origin, where $s = \epsilon e^{j\phi}$

$$\lim_{\varepsilon\to 0} L(j\omega) = \lim_{\varepsilon\to 0} \left|\frac{K}{\varepsilon^2}\right| e^{-2j\phi}$$

$$-\pi/2 \leq \phi \leq \pi/2$$



Stability and Nyquist Criterion

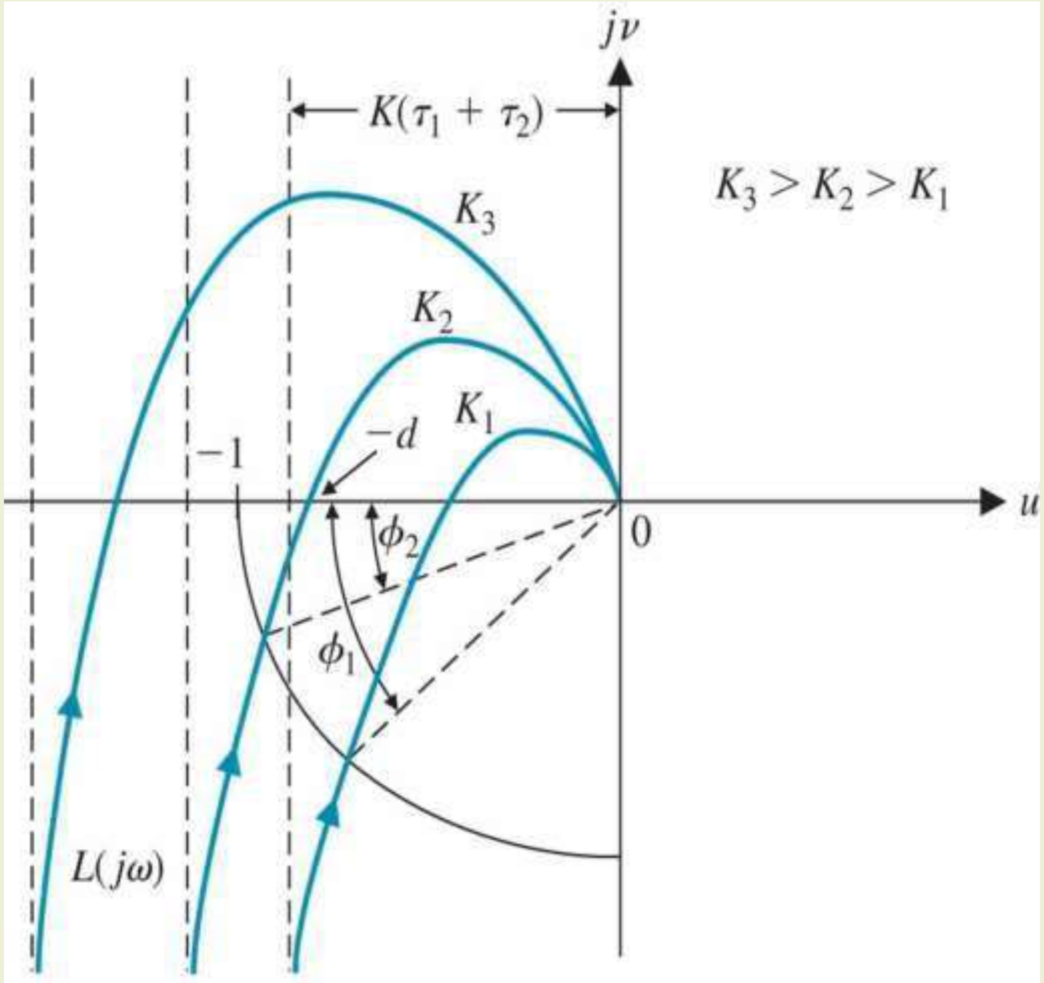
- Relative stability related with relative settling time of each root or pair of roots.
- System with a shorter settling time is considered more relatively stable.

- Nyquist Criterion provides suitable information concerning absolute stability and relative stability

1. Focussing on (-1,0) point on the polar plot, 0 dB and $-180°$ point on Bode diagram
2. Clearly, $L(j\omega)$ locus to this stability point is a measure of relative stability. [Gain Margin and Phase Margin]

Stability and Nyquist Criterion

- Consider $L(j\omega) = \frac{K}{j\omega(j\omega\tau_1+1)(j\omega\tau_2+1)}$



- As K increases, polar plot approaches the (-1,0) point. Eventually, encircles the (-1,0) point for a gain K_3.
- Locus intersects the u-axis at a point $u = \frac{-K\tau_1\tau_2}{\tau_1+\tau_2}$ (c.f Example 9.5)
- It means that system has roots on $j\omega$-axis when $u = -1$.
- As K decreased below this marginal value, stability increased. Margin between $K = \frac{\tau_1+\tau_2}{\tau_1\tau_2}$ and $K = K_2$ is the relative stability[Gain Margin]
- Reciprocal of the gain $|L(j\omega)|$ at the frequency at which the phase angle reaches $-180°$.

Stability and Nyquist Criterion(Conti.)

- Consider $L(j\omega) = \frac{K}{j\omega(j\omega\tau_1+1)(j\omega\tau_2+1)}$

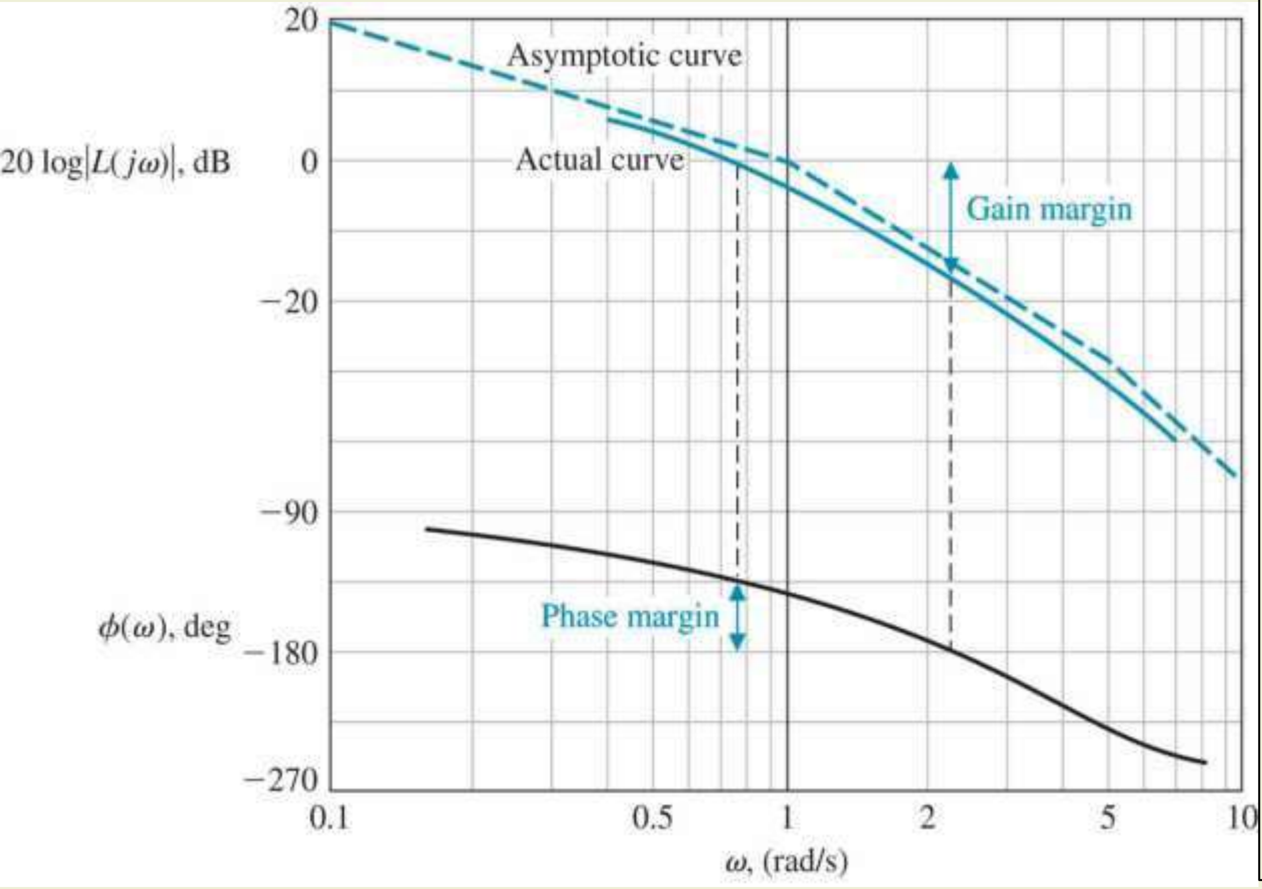
- Gain Margin is the factor by which the system gain would have to be increased for the $L(j\omega)$ locus to pass through the u = -1.
- For gain $K = K_2$, gain margin is equal to the reciprocal of $L(j\omega)$ when *v* =0.
- $\omega = 1/\sqrt{\tau_1\tau_2}$ when the phase shift is $-180°$, we have a gain margin equal to

$$\frac{1}{|L(j\omega)|} = \left[\frac{K\tau_1\tau_2}{\tau_1+\tau_2}\right]^{-1} = \frac{1}{d}, \; 20\log\frac{1}{d} = -20\log d \;\; dB$$

- The Gain Margin is the increase in the system gain when phase = $-180°$ that will result in a marginally stable system with intersection of the -1+*j*0 point on the Nyquist diagram.
- The Phase Margin is the amount of phase shift of the $L(j\omega)$ at unity magnitude that will result in a marginally stable system with intersection of the -1+*j*0 point on the Nyquist diagram.

Relationship Gain and Phase Margin

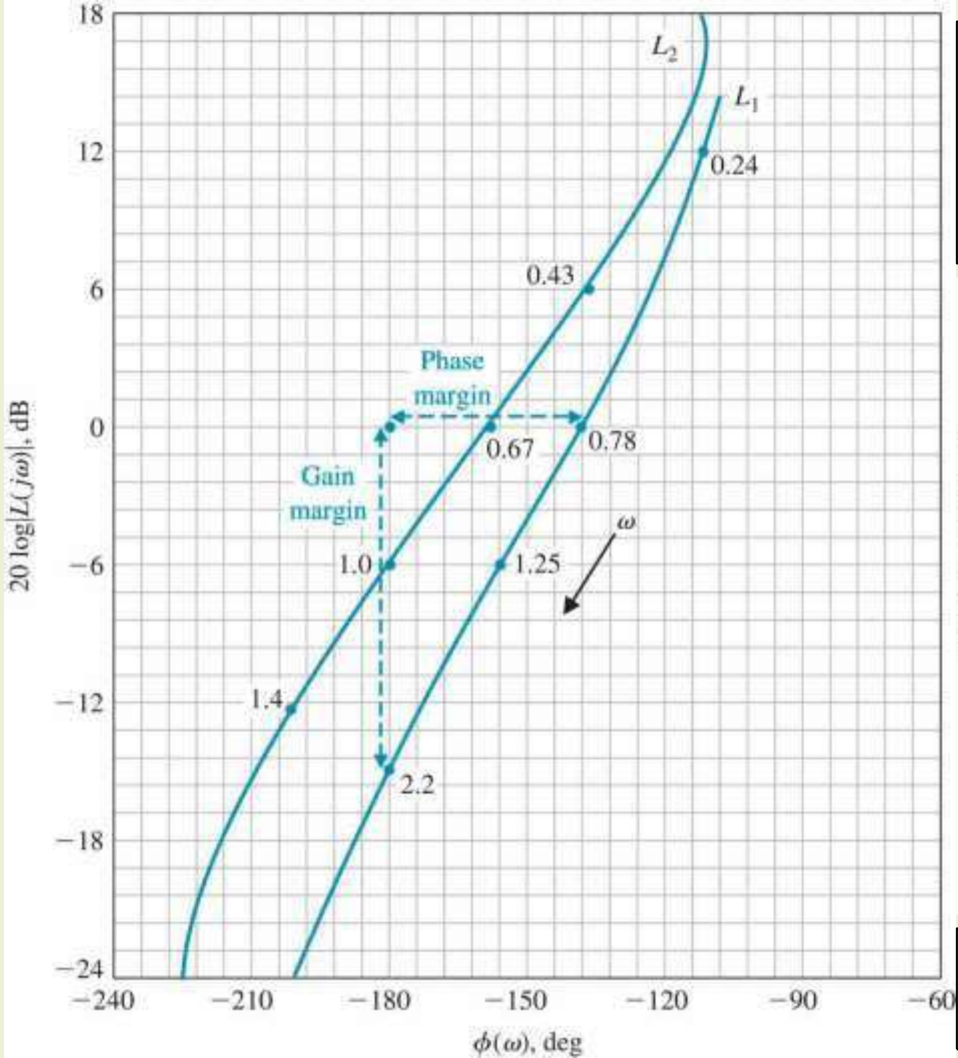
- Critical point (-1,0) point in the $L(j\omega)$ −plane is equivalent to 0 dB and 180° on Bode diagram
- Consider $L(j\omega) = \frac{1}{j\omega(j\omega+1)(0.2j\omega+1)}$



- The phase angle when 0 dB is equal to $-137°$
- Thus phase margin is $180° - 137° = 43°$
- Magnitude when the phase angle is $-180°$ is -15 dB
- Therefore gain margin is 15 dB
- $Magnitude - Phase\ diagram$ is followed.

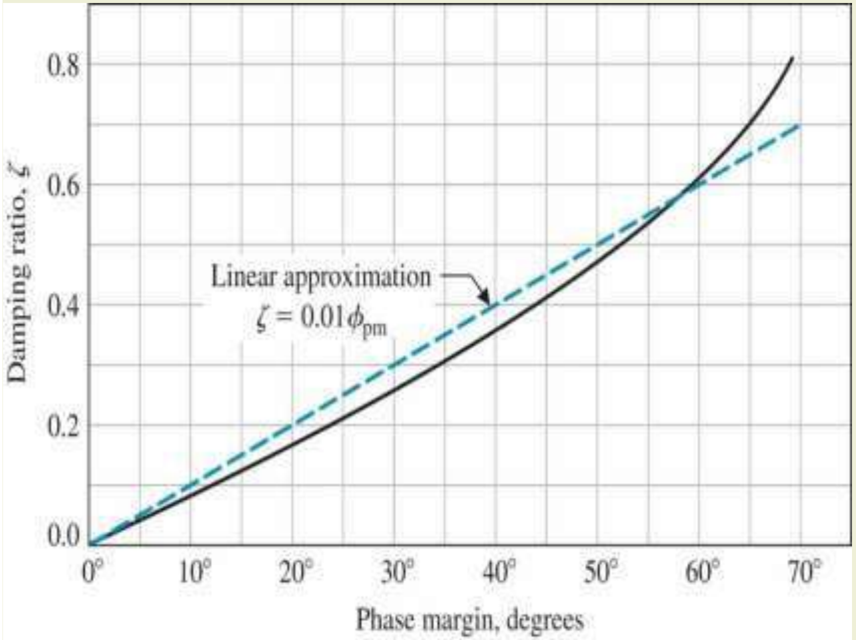
Relationship Gain and Phase Margin

- $L_1(j\omega) = \frac{1}{j\omega(j\omega+1)(0.2j\omega+1)}$, phase margin is 43°, gain margin is 15 dB
- $L_2(j\omega) = \frac{1}{j\omega(j\omega+1)^2}$, phase margin is 20°, gain margin is 5.7 dB



$L_1(j\omega)$ is more stable than $L_2(j\omega)$, it is true

Then, HOW MUCH ? ➔ later talk



Relation between phase margin and damping ratio

Time domain and Frequency domain Criteria

- In Unity feedback case, relation between maximum magnitude and frequency ω_r determined by M circle.
- For $T(j\omega) = \frac{G_c(j\omega)G(j\omega)}{1+G_c(j\omega)G(j\omega)}$, let $G_c(j\omega)G(j\omega) = u + jv$, *then* $M(\omega) = \left|\frac{G_c(j\omega)G(j\omega)}{1+G_c(j\omega)G(j\omega)}\right| = \left|\frac{u+jv}{1+u+jv}\right|$

- Squaring and arranging

$$(1 - M^2)u^2 + (1 - M^2)v^2 - 2M^2u = M^2$$

Finally, $\left(u - \frac{M^2}{1-M^2}\right)^2 + v^2 = \left(\frac{M}{1-M^2}\right)^2$, centered at $u = \frac{M^2}{1-M^2}$ and $v = 0$

- Several constant M circles. Left of u=−1/2 are for M>1, and the circle to the right of u=−1/2 are for M<1.

- When M = 1, the circle es the straight line u=−1/2 .