Structural Mechanics

12.5 Free vibration of two-DOF system

(两个自由度体系的自由振动)

内容(Contents):

1. **概念(Concept)：振型（Mode shape），振型正交性。**
2. **理论(Theory)：牛顿第二定律，和线性方程组的求解**
3. **应用(Application)：Whipping effect “鞭梢效应”**

要求(Requirements):

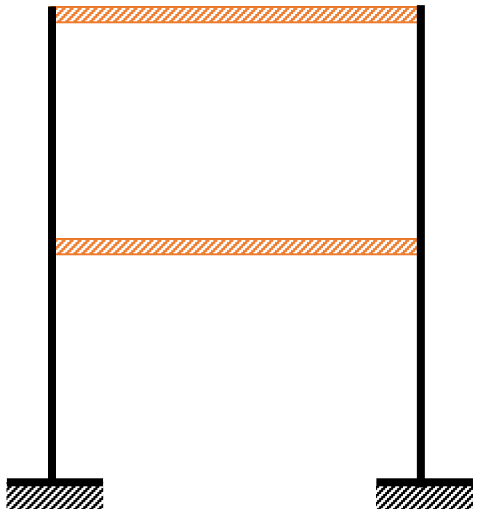
运动方程的建立（刚度法和柔度法）； 熟练频率和振型的求法；理解振型正交性的物理意义。

作业(Homework): 10-17, 19, 20。

12.5 Free vibration of two-DOF system

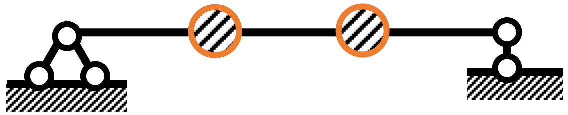
(1) The system have to be simplified into double or multi-DOF system

Multi-story building, bent frame with unequal height



(2) In order to obtain more accurate result

Chimney, high-rise building



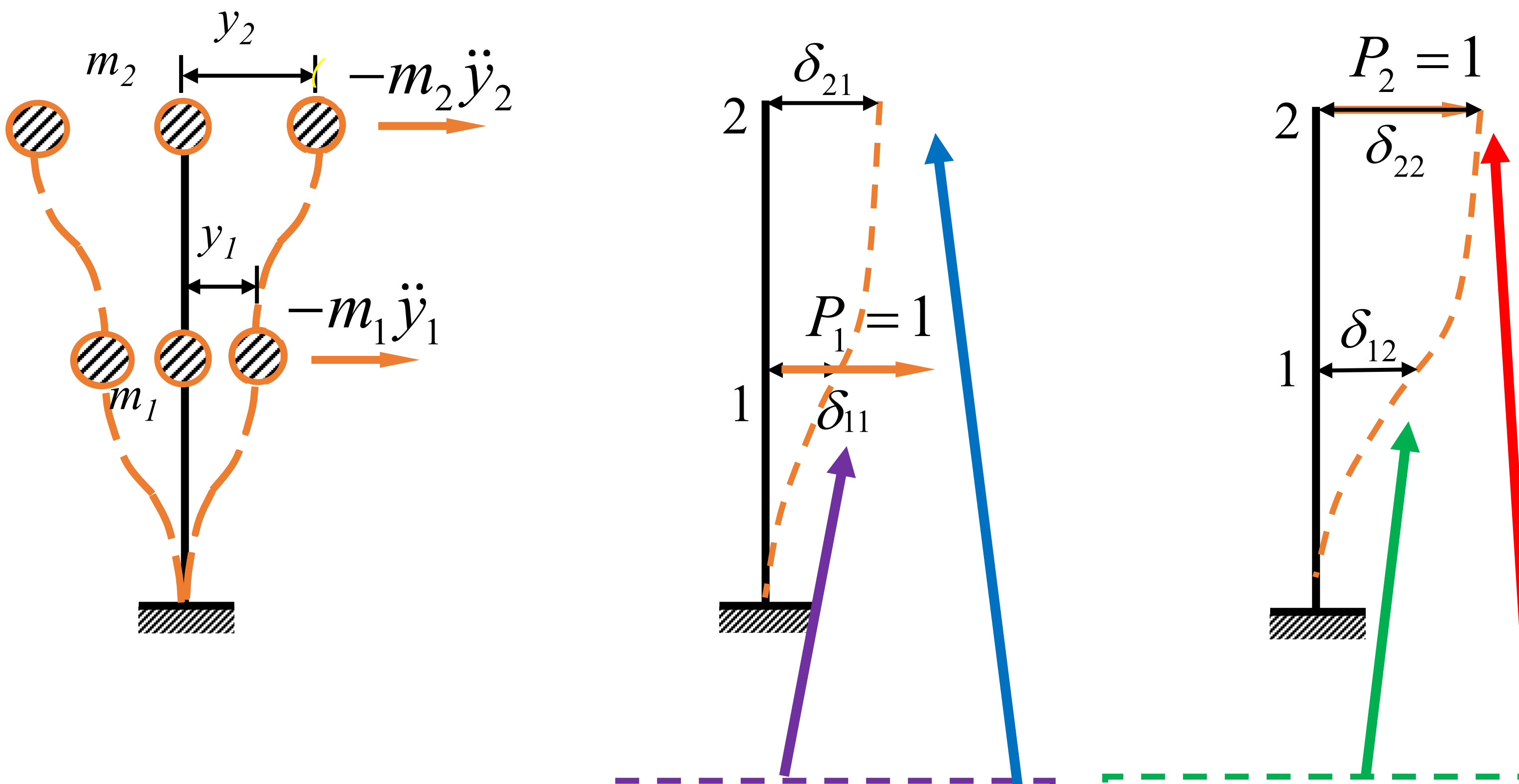
Fundamental methods:

Stiffness method: Based on the force equilibrium equations

Flexibility method: Based on the displacement compatibility condition

(1) Flexibility method

flexibility coefficient： δ

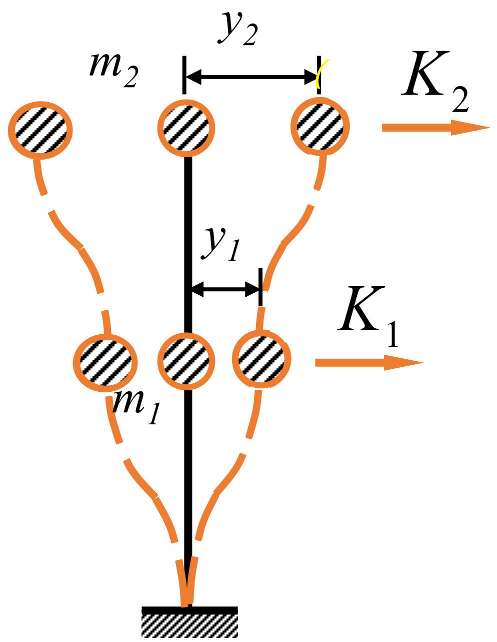


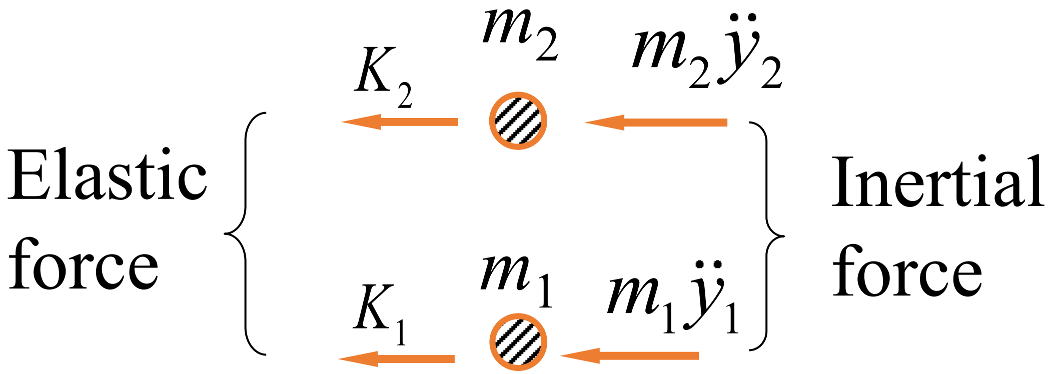
To establish equations：

$$y_1(t)=\delta_{11}[-m_1\ddot{y}_1(t)]+\delta_{12}[-m_2\ddot{y}_2(t)]$$

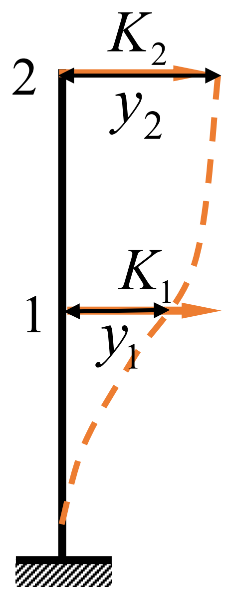
$$y_2(t)=\delta_{21}[-m_1\ddot{y}_1(t)]+\delta_{22}[-m_2\ddot{y}_2(t)]$$

(2) Stiffness method





Mass point isolator



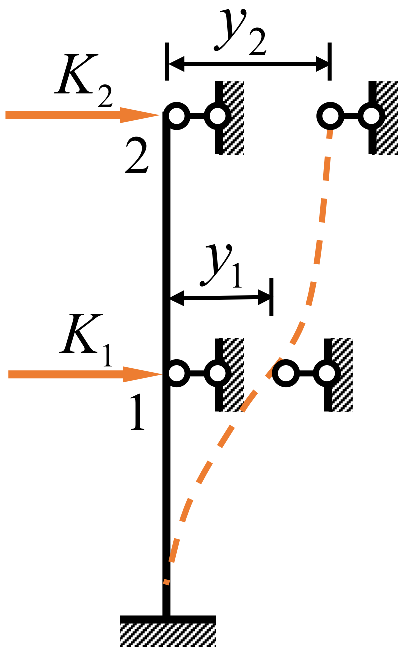
To establish equations:

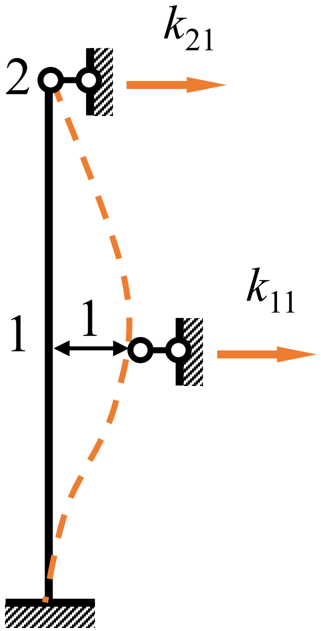
$$m_1\ddot{y}_1(t)+K_1=0$$

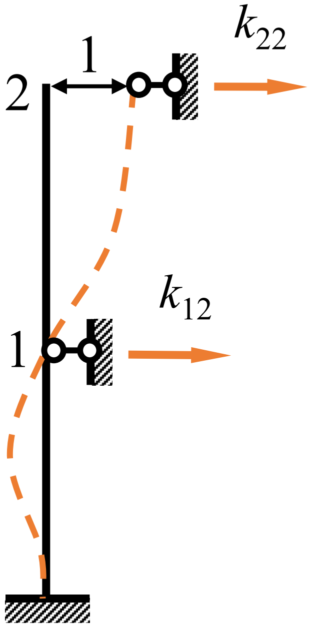
$$m_2\ddot{y}_2(t)+K_2=0$$

How to determine?

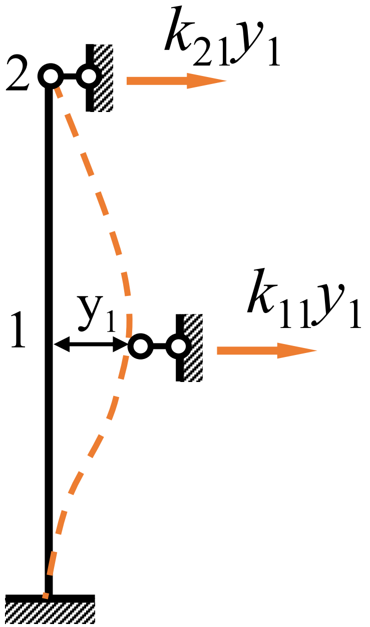
Stiffness coefficient: *k*

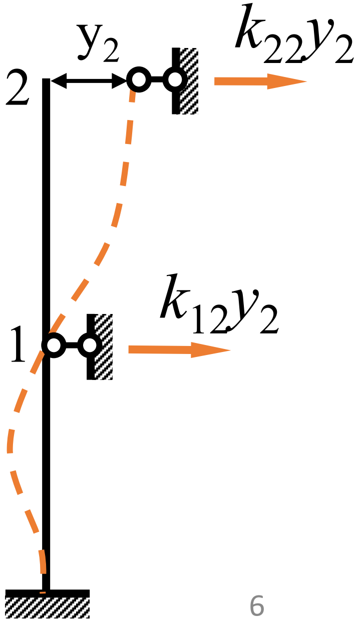






$$\begin{cases} K_1 = k_{11}y_1 + k_{12}y_2 \\ K_2 = k_{21}y_1 + k_{22}y_2 \end{cases}$$





Stiffness coefficient: k

$$\begin{cases} K_1 = k_{11}y_1 + k_{12}y_2 \\ K_2 = k_{21}y_1 + k_{22}y_2 \end{cases} \Rightarrow \begin{cases} m_1\ddot{y}_1(t) + K_1 = 0 \\ m_2\ddot{y}_2(t) + K_2 = 0 \end{cases}$$

Therefore:

$$m_1\ddot{y}_1(t) + k_{11}y_1 + k_{12}y_2 = 0$$

$$m_2\ddot{y}_2(t) + k_{21}y_1 + k_{22}y_2 = 0$$